Fourier Analysis: Creating A "Virtual Laboratory" Using Computer Simulation

Jeff Butterfield, Ph.D.
Western Kentucky University USA
Jeff.Butterfield@wku.edu

&

Norman Pendegraft, Ph.D.
University of Idaho USA
Norman@uidaho.edu

ABSTRACT

At times the desire for specialized laboratory apparatus to support class activities outstrips the available resources. When this is the case the instructor must look for creative alternatives to help meet the desired objectives. This report examines how a virtual laboratory *was created to model and analyze high-speed networking signals in a LAN class using a spreadsheet simulation. The students were able to printout various waveforms (e.g., signals of different frequencies/network media) that are similar to output from test equipment that would have otherwise been cost prohibitive. The activity proved to be valuable in helping students to understand an otherwise difficult concept that is central to modern networking applications. Such simulation is not limited to network signals, but may be applicable in many situations where the artifact under study may be described mathematically.*

Keywords: Simulation; Spreadsheet Modeling; Fourier Analysis; Network Signals; Virtual Laboratory

Introduction

As the 21st century approaches, businesses continue to demonstrate their near-insatiable demand for newer computer technology. Annual investment in hardware and software products is estimated to be $350 billion [1996 figures] (Maddox, 1997). This continued growth has created tremendous job opportunities for MIS graduates (Hube, 1996) and put enormous pressure on colleges and universities to not only keep their IS programs up to date, but to expand them as new applications for technology are developed.

The costs associated with "keeping up" can be formidable. Gone are the days of decades past when a fledgling IS program could get by with some dumb terminals, access to a PDP-11, and perhaps a small cluster of Apple II computers. Learning to program in the languages of the day (e.g., FORTRAN, COBOL, Assembly) and some familiarity with the common operating systems created a marketable MIS graduate. Today, a technologically conscious school (with a bottomless endowment) could build specialized laboratories to support activities in Multimedia, Networking, CASE, Collaborative Technology, CAD/CAM and so on. Sadly, most institutions cannot afford such an array of facilities and are typically limited to clusters of general-purpose personal computers. Instructors that wish to develop learning activities beyond what these general-purpose clusters can support often must look for creative solutions to their problems.

This paper reports on efforts to create a "*virtual*" computer laboratory where specialized activities can be simulated using general-purpose facilities. Specifically, this report will discuss how spreadsheet packages may be used to simulate digital signals and how common Local-Area Network (LAN) media might affect them. However, there are numerous other possibilities that this approach may be adapted to and the goal of this report is to provide an example of how learning activities may be developed or enhanced through computer modeling and simulation that might otherwise not be possible.

Problem

The example used in this report was originally developed for use in a senior-level LAN class in a business-based MIS program. The particular class is applied in nature and the students are required to complete weekly lab activities in addition to their normal class work. Budget limitations dictated that only inexpensive Ethernet cards be used. Compatible with the IEEE 802.3 standard these network cards operate at transfer rates of 10 Mega-bits/second. When originally developed this 10 Mbit rate was consistent with the speed of the popular computers of the day. However, as the speed of desktop computers has jumped from tens to hundreds of MHz in recent years the older Ethernet standard has become a bottleneck for many network users. 100 Mbit technology, so-called Fast Ethernet, is increasingly available, but its higher price is still a matter of concern. In addition to the higher hardware costs, the migration to Fast Ethernet also requires improved wiring, which adds its own set of costs to the equation.

One of the most common questions raised in the class is why the speed of an older Ethernet card cannot simply be increased (i.e., add a faster clock chip). The usual follow-up question is why the network media must also be upgraded when Fast Ethernet cards are introduced. As there is no simple answer to the questions a common response might be some short discussion about bandwidth along with the hope that the students would be satisfied and that the lecture could proceed. However, most students recognize that the coaxial cable used in LAN applications is similar to that used to carry 100 cable television stations into their homes and apartments and the simple answer was usually unsatisfactory.

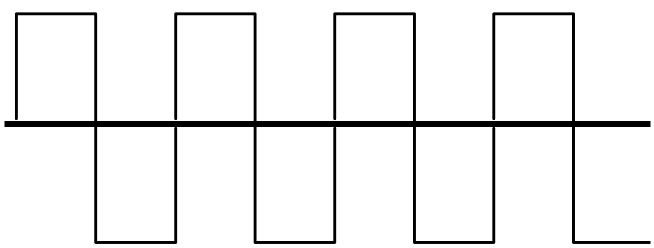

Figure 1 - Polar Synchronous Signal

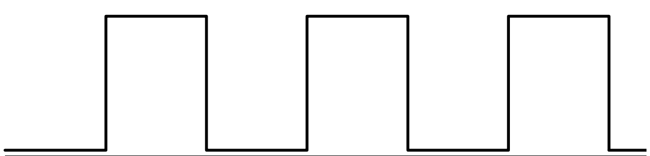

Figure 2 - Unipolar Signal

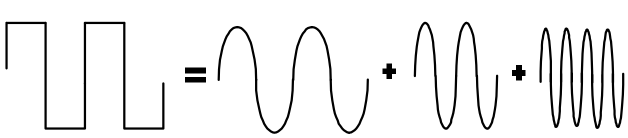

Figure 3 - Fundamental + Harmonics

At the root of such questions lies the broader question of how are digital signals affected when their speed is pressed into the VHF and UHF ranges. The answer to the question is significant and is becoming a serious issue that threatens to hinder improvements in computer and telecommunications speed. The next section provides some background on the nature of digital signals and introduces the computer simulation that was used to give the students some exposure to these principles.

Background

In most computer systems a binary digital signal will consist of a series of bits that is represented by some voltage or current waveform. This commonly takes the form of a square wave that fluctuates between two different states (e.g., 1 or 0) as represented by two different voltage levels. Traditional transistor logic circuits vary between 0v and 5v (also known as unipolar synchronous wave) and are shown in figure 1. LAN circuits more commonly use a polar synchronous wave where a binary 1 is represented as a +V pulse while a binary 0 is represented as a -V pulse as shown in figure 2. The latter is more efficient and better suited to the longer distances that a LAN signal would be expected to travel.

Square waves are preferred over common sine waves in that the sharp rising and falling edges allow for better timing. The transition between a 0 and 1 state is very fast compared to the period of the wave. In a sine wave there is no clear distinction between high and low voltages. A semi-conductor made by one process might *trigger* when the sine wave reaches 50% of its maximum voltage. Another might *trigger* at 65%. If this were the case the output of any logic circuit would be distorted significantly. However, the use of square waves is not without its problems.

The difficulties inherent in sending square waves at high frequencies lies in their complexity. Any non-sinusoidal, single-valued waveform is actually made up of an infinite number of sine waves and/or cosine waves. The fundamental sine/cosine wave is the same frequency as that of the square wave. The other waves are harmonics (i.e., multiples) of this fundamental frequency. In particular, a 10Mbit digital signal is actually made up of a 10MHz fundamental sine wave and harmonics at 30MHz, 50Mhz, 70Mhz and so on, as shown in figure 3.

The traditional method for analyzing complex waveforms such as digital square waves has been mathematically by a process called Fourier Analysis (Kreysig, 1967). This method is named after Jean Baptiste Fourier who, in 1822, developed some of the basic mathematical concepts now used in signal analysis. Using Fourier Analysis an equation that describes a particular waveform in terms of the time domain may transformed into an equation that describes the same waveform in the frequency domain. The general formula is given as:

$$f(x) = \sum_{n=1}^{\infty}\left[a_n \sin\left(\frac{nx\pi}{L}\right) + b_m \cos\left(\frac{nx\pi}{L}\right)\right]$$

where the value of a_n and b_m can be easily calculated. For a simple square wave the formula may be simplified to:

$$\text{F(t)} = \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{nt\pi}{L}\right)$$

where L is the wavelength of the fundamental frequency and C is some constant.

However, this model assumes that the signal is travelling through an ideal transmission media (e.g., no losses). In real media the higher frequency components are attenuated. To capture this an attenuation factor, a_n, is added to each term of the equation as:

$$\text{F(t)} = \sum_{n=1,3,5\ldots}^{\infty} \frac{a_n}{n} \sin\left(\frac{nt\pi}{L}\right)$$

The spreadsheet is then used to calculate the various terms for several values of ***t*** to create the waveform simulation. The spreadsheet is used because for most business undergraduates, the analysis is beyond their capabilities to do manually. The spreadsheet also shifts the focus from one of computation to actual exploration of the phenomena. This alternative method of analyzing the waveform graphically simulates the waveform by adding the appropriate sine-wave components calculated using the above formula. The sum is plotted and the resultant waveform is displayed. This approach is more manageable from a mathematical perspective and forms the basis of the “virtual” laboratory simulation.

Limits of LAN Media

The fact that a square wave is actually composed of many different waves each a harmonic at some multiple of the fundamental frequency by itself is not the issue. Assuming that network media with unlimited bandwidth was available there would be little problem. Unfortunately, the two most common types of network transmission lines, coaxial cable and twisted pair, both have bandwidth limitations. Coaxial cable was the media originally called for in the IEEE standard. It is still in use, although unshielded twisted pair has become a less expensive alternative.

Since each conductor in the transmission line has a certain length and diameter, it must have a certain amount of resistance and inductance. Because the two conductors are close to each other there must also be a certain amount of capacitance present. At high frequencies the effects of the inductance and capacitance are much larger than the resistance of the wire and as such the resistance may be ignored. The equivalent circuit representation for a length of transmission line is shown in figure 4.

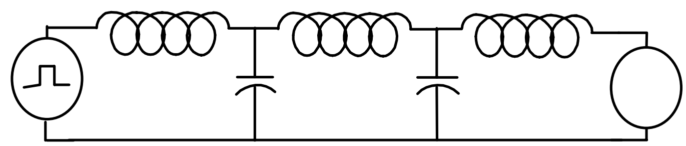

Figure 4 – Transmission Line Equivalent Circuit

The inductance in the line serves to resist or restrict the flow of the signal through it. This effect becomes more pronounced the higher the frequency of the signal. The capacitance in the line provides an *alternative* path (i.e., short-circuit) through it. This effect also becomes more pronounced the higher the frequency of the signal.

When used at the frequencies common to modern computers a length of coaxial cable acts as a “low-pass” filter that allows the lower-frequency components of the square wave to pass while restricting the flow of the higher-frequency harmonics. As a general rule, the higher the harmonic the lower its relative amplitude, so that the highest harmonics are often ignored when calculating required bandwidth. However, to maintain the integrity of modern high-speed signals the media needs to support a bandwidth of 10 times the bit rate (Schweber, 1991). Why this is so may be inferred from our simulator and is part of the second exercise described below.

Virtual Laboratory Experiment

The goal of the learning exercise was to have the students experiment with a length of network media and to see what happened as the media was expected to carry higher-frequency signals. As the necessary test equipment was not available the students simulated a simple square wave traveling through a piece of RG-58 coaxial cable and common twisted-pair network cable.

The characteristics of these two media were derived from manufacturer's specifications. Of principal interest in the degree to which signals are attenuated in the cables of specified length. For most such media, there is a linear relationship between the log of the frequency and the log of the attenuation as measured in db (see figure 5).

The relationship can be mathematically expressed as:

$\log A = b + m \log F$

which, solving for A yields:

$A = F^{m}10^{b}$

Where:
- A = attenuation
- m = slope of the line on log-log graph
- b = y-intercept of the line on log-log graph
- F = frequency of the signal

We use these equations to estimate the attenuation of each harmonic in the square-wave signal. However, as attenuation (A) is commonly measured in decibels (db) we must convert these values to an attenuation factor (the fraction of the original signal that is ultimately lost in transit):

$a = 10^{-A/20}$

Designing the Virtual Laboratory

To create the laboratory simulation a basic spreadsheet is used to calculate and graph the Fourier elements of the square wave. By entering media-specific parameters (readily available from manufacturer's catalogs or corresponding Internet sites), one can use a single spreadsheet to simulate several different media carrying digital signals of varying frequencies.

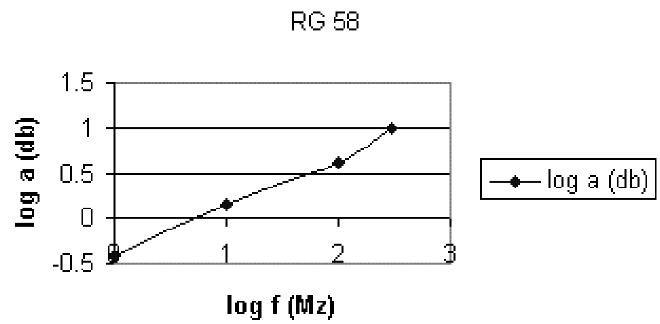

Figure 6 - Coaxial Cable Attenuation

The values for m and b are entered in the spreadsheet in cells B5 and B6 (see figure 6). The spreadsheet uses a two-dimensional array (D15..I114) to calculate the terms in the

	A	B	C	D	E	F	G	H	I
1	Square Wave: RG 58								
2	Fundamental (MHz)		Harmonic	1	3	5	7	9	11
3		1	Frequency	1	3	5	7	9	11
4									
5	m	0.56	db (calc)	0.40	0.74	0.98	1.18	1.36	1.52
6	b	-0.4	Attenuation Factor	0.9552	0.9187	0.89326	0.8726	0.85481	0.83901
7									
8			Include	1	1	1	1	1	1
9			Weight	0.9552	0.9187	0.89326	0.8726	0.85481	0.83901
10		4 / pi =	1.273239545						
11									
12			n	1	3	5	7	9	11
13			1 / n	1	0.33333	0.2	0.14286	0.11111	0.09091
14	T	t	Real RG58						
15	100	0	0	0	0	0	0	0	0
16		1	0.413997994	0.06279	0.06246	0.0618	0.06083	0.05954	0.05795
17		2	0.75677618	0.12533	0.12271	0.11756	0.11007	0.10054	0.0893
18		3	0.979651104	0.18738	0.17861	0.1618	0.13837	0.11023	0.07966

Figure 5 - Spreadsheet Simulation

Fourier series. The vertical axis is time and the horizontal axis is frequency. Each cell contains a formula to calculate the Fourier term for the given frequency at the specified time.

Cell B3 contains the fundamental frequency in MHz. By changing this value we can simulate the performance of the medium at any desired frequency. Attenuation factors are calculated in rows 5 and 6 as functions of the harmonic frequency (row 3). Row 8 is a user-entered value (1 or 0) that allows the user to include or exclude a harmonic of the signal. The product of the attenuation factor and the indicator variable is calculated in row 9. These weights are then used to calculate the terms of the Fourier series using the SUMPRODUCT function in column C of the spreadsheet. Omitting the attenuation factor (or setting it equal to 1 for all frequencies) permits us to model an ideal cable. An ideal cable is defined as one that exhibits no losses at any frequency. It was included to provide a more realistic view of the losses that real network media exhibit. It should be noted that both simulations are approximate as the actual waveform is made up of an infinite series while this model is eventually truncated.

It should be noted that this particular spreadsheet is only one possible way in which this activity could be done. In a class where the students have a reasonable level of competence with spreadsheet modeling it would be advantageous to give them the data and have them develop their own simulations.

Working With the Virtual Laboratory

To illustrate the model's use and show how it might be used to supplement a class activity we offer several graphs. Figures 8-10 respectively show the sum of the fundamental and first harmonic, fundamental and first two harmonics, and the fundamental and first five harmonics. As is evidenced, the sum of the individual waveforms evolves quickly into the traditional square-wave used in digital electronics. We have used this first *wave-building* exercise as part of a networking-class lecture to help students visualize the components of a digital signal and help them to better understand the Fourier concept. By starting with the fundamental frequency and adding one harmonic at a time they are able to see how the Fourier components contribute to the complex waveform. This understanding is essential for the subsequent activities.

Figure 7 shows a signal made up of the fundamental and a single harmonic. While it approximates the square wave (also shown), the gradual transitions between states could create potential problems in timing (i.e., at what point does the rising signal switch from a 0 to a 1). As additional harmonics are added, as in figures 8-9, the resulting waveform more closely resembles the intended square wave.

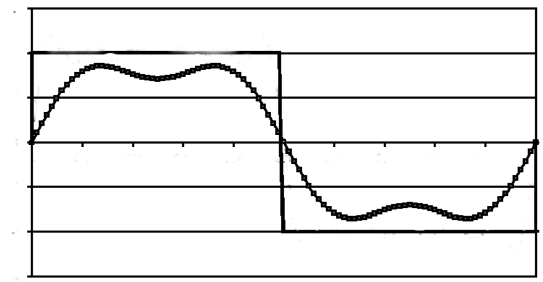

Figure 8. Fundamental + 1 Harmonic

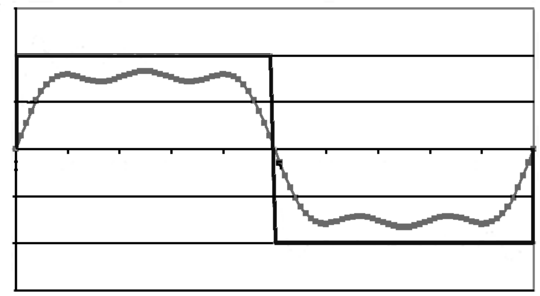

Figure 7 - Fundamental + 2 Harmonics

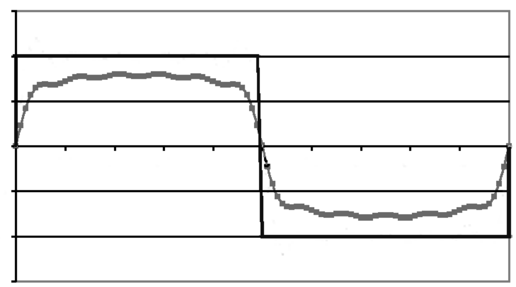

Figure 9 - Fundamental + 5 Harmonics

In a second learning exercise, we evaluate the impact of frequency truncation on a transmission line. One familiar example of a transmission media with a constrained bandwidth is the common telephone line. It has a bandwidth of approximately 3000 Hz. Assuming that a 100 Hz. square-wave were transmitted over such a line the media would be able to carry the fundamental and the first five harmonics. Figure 9 shows that a square-wave is recognizable with only some loss in intelligibility. However, if the fundamental frequency were 1000 Hz. then the voice-grade line would only be able to transmit the fundamental and one harmonic (as shown in figure 7).

The use of a telephone line in this example has proven effective for several reasons. Students are familiar with such lines and their inherent lack of bandwidth. Many have had experience connecting computers using modems over a telephone line and this activity is a logical introduction to a discussion on how high-speed modems function over such low-grade lines. If the students are not familiar with the bandwidth limitations of a telephone line a simple activity in which they attempt to transmit high-fidelity music over a common voice line and report on the high-frequency response on the receiving end often proves insightful.

Figure 9 also shows why a bandwidth of 10F is considered the minimum acceptable. The 5th harmonic represents a frequency 11 times the fundamental. Thus, a bandwidth of 10 F will permit a recognizable square wave to be transmitted.

The third activity explores the impact of attenuation in transmission lines. Using a step-wise comparison, the higher harmonics of the signal can then be attenuated with the resulting received signal being displayed. Figures 10-13 show the results of such a comparison over a range of frequencies from 100 kHz to 1 Ghz. The results show that as the frequencies increase, the signal becomes so attenuated that it may no longer be useful.

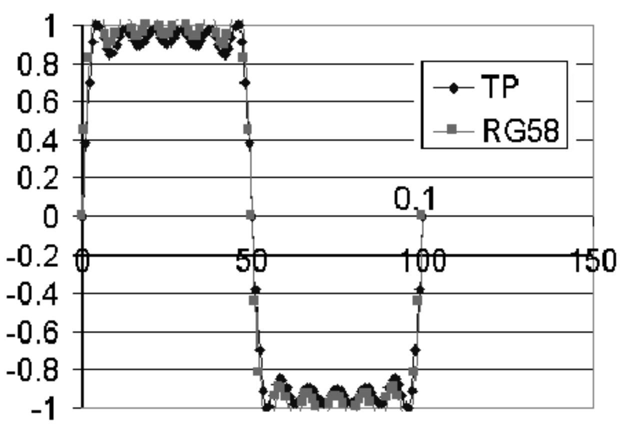

Figure 10 - Twisted Pair vs. Coax at 100Khz

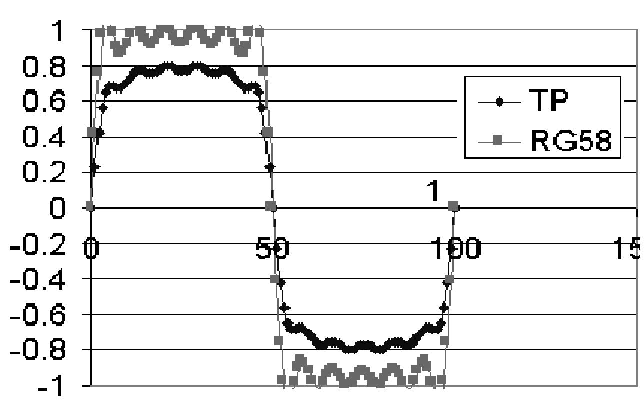

Figure 11 - Twisted pair vs. Coax at 1 Mhz

The fourth activity provides for a side-by-side comparison of two different types of network media to see how each responds to a similar signal. In this case the traditional coaxial cable is compared to the more common twisted-pair (UTP). The above figures 10-13 show the differences not only by frequency, but also by media type at each frequency. At 100 Khz there is very little signal degradation in either media. However, differences in capacity become evident as the frequency increases. At 10 MHz (the common Ethernet standard) coaxial cable maintains the integrity of the square wave much better than the twisted pair. By 100 MHz (the Fast-*Ethernet* standard) the single twisted pair is all but useless.

This activity is valuable in that it demonstrates how particular media perform in different circumstances (e.g., long cable runs – which may be simulated with slight modifications to the spreadsheet model). The differences in the media characteristics between coaxial cable and twisted pair are clearly evident and make the point stronger than would a similar explanation in a textbook. This activity is extremely helpful in explaining why the coaxial cable that was originally called for in the IEEE Ethernet standard cannot be used on the faster 100-Base-T standard (100 MHz). It also helps to answer the question about why twisted-pair segments are limited to 100 meters while coaxial cable segments may be as long as 175 meters (signals in UTP degrade more quickly).

Measuring Simulation Effectiveness

The pedagogical value of a simulation of this sort would be minimal if it failed to teach the concepts that it was designed to present. Its value would also be diminished if it were little more effective than other less-involved teaching methods. As this material was presented to different classes, some using a traditional lecture format and others using the simulation to supplement the lecture, the opportunity for comparisons existed.

One possible approach would be to create a set of pre- and post-tests over the subject material and attempt to gauge improvements in understanding. While methodologically appealing, such an approach was judged potentially threatening to the student subjects. Taking what would appear to be multiple examinations on a single topic area would violate (Kirk, 1982) recommendation that such measurements be as subtle as possible.

An alternative testing approach could make use of a routine mid-term examination as a less-conspicuous instrument. This idea was also judged to be impractical as the number of questions that would be required to validly measure this topic would unfairly skew the exam in favor of this one concept. As this effort is largely exploratory in nature it was decided that allowing the subjects to self-report the degree to which they felt that they understood the material and their level of satisfaction with the pedagogical approach. While qualitative measures of this sort have their shortcomings they can provide sufficient information to know if the simulation was effective (Cook & Campbell, 1979).

The data were taken from a supplemental end-of-class survey. This survey was administered in addition to the formal university class survey and solicited more specific feedback about course topics and issues. Two questions were posed to all of the participants while a third question was directed only at those that made use of the virtual laboratory. The two common questions were asked for each of the major lecture topics in the class.

CLASS TOPIC: High-speed digital signals, bandwidth and limitations of traditional network media

Q1: How well do you feel that you understand the concepts presented in this class segment?

Q2: How effectively did the instructor present the material for this class segment?

Q3: To what extent do you feel that the spreadsheet simulation helped you understand the concepts presented in this class segment?

	Question #1	Question #2	Question #3
Virtual Lab	5.0	5.2	6.3
Lecture Only	2.9	4.7	n/a

Table 1 - Survey Results

The mean responses to each of these questions are shown in table 1. The responses were measured on a 7-point Likert scale ranging from Not at All to Extremely Well. No test of significance is offered as no formal hypothesis was being examined.

The feedback suggests that the subjects making use of the virtual laboratory felt that they understood the concepts better than those that did not. There was a much smaller difference of opinion on the effectiveness of the class instruction[1].

[1] The quality of the instructor's teaching skills were not specifically controlled for. However, it should be noted on the university survey the instructor scored well above average for teaching effectiveness in the class (3.7/ 4.0). This would suggest that the simulation is not merely masking poor classroom ability.

On the question of the usefulness of the virtual laboratory in helping the students better understand the material there was a decidedly positive response. Two reasons for this surfaced in the optional written feedback that was part of the questionnaire. The first was that the simulation allowed them to "see" or visualize what was happening to the signals. The second

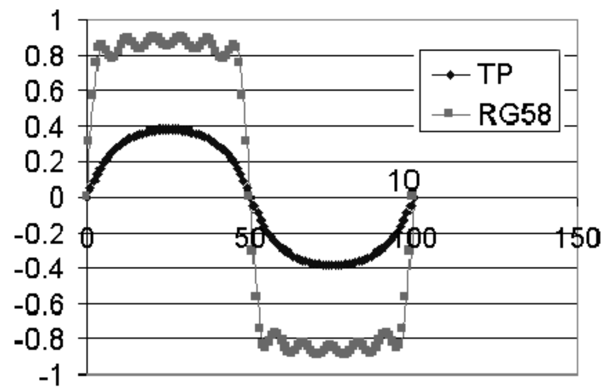

Figure 12 - Twisted Pair vs. Coax at 10 Mhz

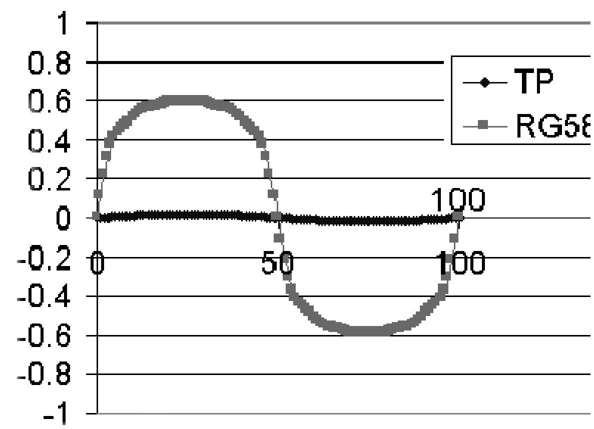

Figure 13 - Twisted Pair vs. Coax at 100 Mhz

was that the what-if ability of the spreadsheet allowed them to try signals of different frequencies over and over until they felt like they understood it. The abilities to visualize complex phenomena and to go over difficult material as often as each individual student requires are not often associated with traditional lecture style of instruction.

Conclusion

This report provides an example of how using a common spreadsheet program (as is often found in public computing clusters) an instructor can create a virtual laboratory with associated experiments without the investment in expensive equipment or special facilities. This particular simulation allows students to model complex phenomena such as the Fourier Transform of a complex digital signal without being straddled with the associated mathematical analyses. By allowing the spreadsheet to simulate the data the student is able to focus on visualizing what is taking place and what limitations the media imposes on the signal.

The cost to develop and execute such a set of activities is limited to an instructor's time. Further, the simulation need not be limited to digital signals. Many phenomena that can be described mathematically (e.g., accounting systems, assembly lines, etc.) could potentially be simulated and analyzed on a desktop computer in similar fashion. It is hoped that the particular example in this report serves as a catalyst for other instructors to develop other simulation and modeling activities that augment their class topics.

References

ARRL, *The ARRL Handbook*, Newington, CT: American Radio Relay League Press, 1995.

Cook, T.D., and D.T. Campbell, *Quasi-Experimentation: Design and Analysis for Field Settings*, Boston: Houghton Mifflin Co., 1979.

Hube, K., "Top Jobs for 1996," *Money Magazine: Special Forecast Issue*, 1996.

Johnson, D.E., J.L. Hilburn, and P.R. Johnson, *Basic Circuit Analysis*, Englewood Cliffs, CA: Prentice Hall, 1978.

Kirk, R.E., *Experimental Design: Procedures For the Behavioral Sciences*, Monterey, CA: Brooks and Cole Publishing, 1982.

Kreysig, E., *Advanced Engineering Mathematics*, New York: Wiley and Sons, 1967.

Maddox, K., "The New IS Budget Agenda," *Information Week*, April 7, (1997), 56-58.

Goff, L., "Information Technology: Big Major on Campus," *Computerworld*, June 16, (1997), 100.